Drinfeld's Yangian

Plan of the first lectures:

Lecture 1 : definition from the 6-vertex model

Lecture 2 : finite-dimensional representations

Lecture 3, Lecture 4 : algebraic / analytic construction of the universal R-matrix

Lectures 5 - 9 : geometric construction of R [Maulik-Okounkov 2012]

Today:

Lattice Models of Statistical Mechanics

↓

Yang-Baxter equation = Solvability

Rational solution	Trigonometric	Elliptic

RTT algebra of Faddeev-Reshetikhin-Takhtajan

↓	↓	↓
$Y_\hbar(\mathfrak{sl}_2)$	$U_q(L\mathfrak{sl}_2)$	$\mathcal{E}_{\hbar,\tau}(\mathfrak{sl}_2)$

Lecture 1

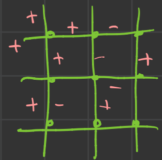
An elementary view towards lattice models

- $M, N \in \mathbb{Z}_{>0}$
- Λ_{MN} = rectangular grid with M rows and N columns

we impose periodic boundary conditions

$\Longrightarrow$ Λ_{MN} is a grid on the torus

- a configuration C is a labelling



$$\text{edges}(\Lambda_{MN}) \longrightarrow \{+,-\}$$

$\Longrightarrow$ around a vertex we have 16 possibilities each with weight $a_i \in \mathbb{C}$ $i = 1, \ldots, 16$

- the weight of C is

$$\mathrm{wt}(C) := \prod_{i=1}^{16} a_i^{m_i} \longleftarrow m_i = \begin{array}{c}\#\text{ vertices} \\ \text{with the } i^{\text{th}} \\ \text{labelling}\end{array}$$

- the partition function is

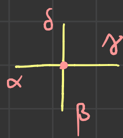
$$Z = \sum_{C \in \mathrm{Config}} \mathrm{wt}(C)$$

The vertex model behind it

atoms in a 2d crystal — vertices in the lattice Λ_{MN}

states of the bonds joining each pair of neighbouring atoms — $+ \quad -$

interacting energy of the atom depends only on bond states — weights $a_i \in \mathbb{C}$

(i) 

$\mathcal{E}(\text{atom}) = \mathcal{E}(i)$ $\rightsquigarrow$ $a_i = \exp(-\beta\, \mathcal{E}(i))$, $\beta = \frac{1}{KT}$

atom with bonds (i) — Boltzmann weights

$$\mathcal{E}(\Lambda\text{-state}) = \sum_{\text{atoms}} \mathcal{E}(\text{atom}) \rightsquigarrow wt(\mathcal{C}) = \prod_i a_i^{m_i} = \exp(-\beta\, \mathcal{E}(\mathcal{C}))$$

$$\mathcal{Z} = \sum_{\Lambda\text{-states}} \exp(-\beta\, \mathcal{E}(\Lambda\text{-state})) = \sum_{\mathcal{C}} wt(\mathcal{C})$$

exactly solvable model $=$ explicit formula for $\mathcal{Z}$

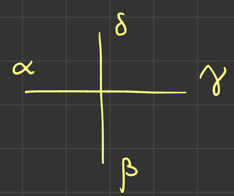
R-matrices and transfer matrices

Goal: understand $\mathcal{Z}$ as a trace

R-matrix: encode $a_1, \dots, a_{16}$ in a matrix

$$R := \left[R(\alpha\beta|\gamma\delta) \right]_{\alpha,\beta,\gamma,\delta \in \{\pm\}}$$

$R(\alpha\beta|\gamma\delta)$ is the weight of

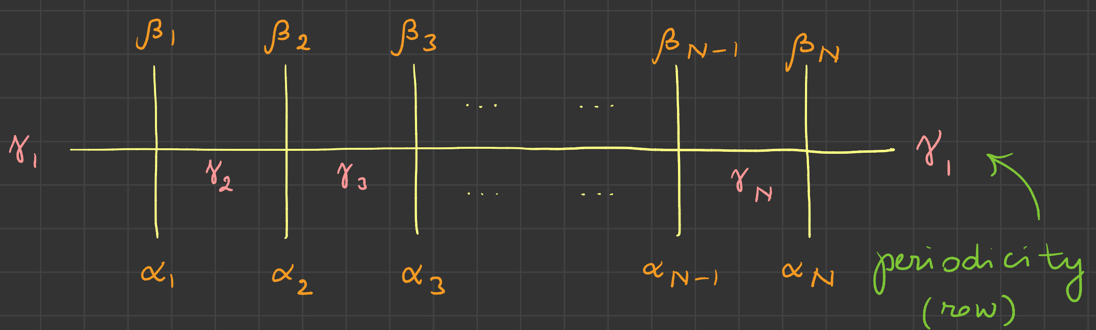


Notation: $\alpha, \beta, \gamma, \dots \in \{\pm\}$

$\underline{\alpha}, \underline{\beta}, \underline{\gamma}, \dots \in \{\pm\}^N$

Transfer matrix: for $\underline{\alpha}, \underline{\beta} \in \{\pm\}^N$

$$T(\underline{\alpha}, \underline{\beta}) := \sum_{\underline{\gamma} \in \{\pm\}^N} R(\gamma_1\alpha_1|\gamma_2\beta_1) \cdot R(\gamma_2\alpha_2|\gamma_3\beta_2) \cdots R(\gamma_N\alpha_N|\gamma_1\beta_N)$$

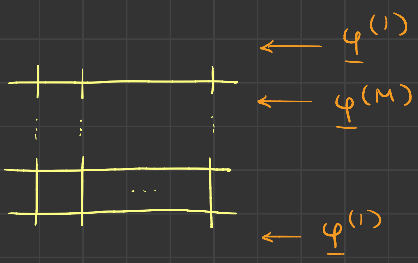


$$\boxed{\begin{array}{l} R \in \mathrm{End}\left(\mathbb{C}^2 \otimes \mathbb{C}^2\right) \\ T \in \mathrm{End}\left((\mathbb{C}^2)^{\otimes N}\right) \end{array}}$$

Lemma $\mathcal{Z} = \mathrm{tr}\left(T^M\right)$

periodicity

pf.



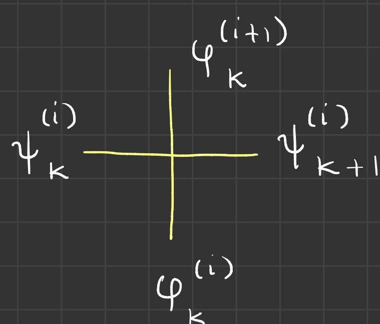
$$\mathrm{tr}\left(T^M\right) = \sum_{\underline{\varphi}^{(1)}, \dots, \underline{\varphi}^{(M)}} T\left(\underline{\varphi}^{(1)} \middle| \underline{\varphi}^{(2)}\right) T\left(\underline{\varphi}^{(2)} \middle| \underline{\varphi}^{(3)}\right) \cdots$$

$$\cdots T\left(\underline{\varphi}^{(M)} \middle| \underline{\varphi}^{(1)}\right)$$

$$= \sum_{\underline{\varphi}^{(1)} \dots \underline{\varphi}^{(M)}} \sum_{\underline{\psi}^{(1)} \dots \underline{\psi}^{(M)}} \prod_{\substack{i = 1 \dots M \\ k = 1 \dots N}} R\left(\psi_k^{(i)} \varphi_k^{(i)} \middle| \psi_{k+1}^{(i)} \varphi_k^{(i+1)}\right)$$

rows of vertical edges

rows of horizontal edges



$\square$

Monodromy matrix

Goal: understand T as a trace

Define $\mathcal{T} \in \mathrm{End}\left(\mathbb{C}_0^2 \otimes (\mathbb{C}^2)^{\otimes N}\right)$ by

$$\mathcal{T}(\gamma\underline{\alpha}|\gamma'\underline{\beta}) := \sum_{\gamma_2,\dots,\gamma_N \in \{\pm\}} R(\gamma\alpha_1|\gamma_2\beta_1)\cdots R(\gamma_N\alpha_N|\gamma'\beta)$$

free bond state

$$\rightsquigarrow \quad \mathcal{T} = R_{01} R_{02} \cdots R_{0N}$$

$$T = \mathrm{tr}_{\mathbb{C}_0^2}(\mathcal{T})$$

$$\mathcal{T}_{N=1} = R_{01}$$

by periodicity

Commuting transfer matrices

T, R = matrices corresponding to $(a_1, \dots, a_{16})$

T', R' = —" ——— " —— $(a_1', \dots, a_{16}')$

Goal: we want necessary and sufficient conditions for $TT' = T'T$

integrability: a large family of commuting operators

$\rightsquigarrow$ asymptotic estimate $Z \sim \lambda_{max}^M$

$$TT' = \mathrm{tr}_{\mathbb{C}^2_0 \otimes \mathbb{C}^2_{\bar{0}}} \left(\mathcal{T}_0 \mathcal{T}'_{\bar{0}} \right)$$

$$T'T = \mathrm{tr}_{\mathbb{C}^2_0 \otimes \mathbb{C}^2_{\bar{0}}} \left(\mathcal{T}'_{\bar{0}} \mathcal{T}_0 \right)$$

If $R''_{0\bar{0}}$ conjugates $\mathcal{T}_0 \mathcal{T}'_{\bar{0}}$ to $\mathcal{T}'_{\bar{0}} \mathcal{T}_0$, we're done,

so it's desirable to have:

$$R''_{0\bar{0}} \mathcal{T}_0 \mathcal{T}'_{\bar{0}} = \mathcal{T}'_{\bar{0}} \mathcal{T}_0 R''_{0\bar{0}}$$

For $N=1$, we get the Yang-Baxter equation

$$R''_{0\bar{0}} R_{01} R'_{\bar{0}1} = R'_{\bar{0}1} R_{01} R''_{0\bar{0}} \qquad \text{(YBE)}$$

Thm: (YBE) $\Longrightarrow TT' = T'T$

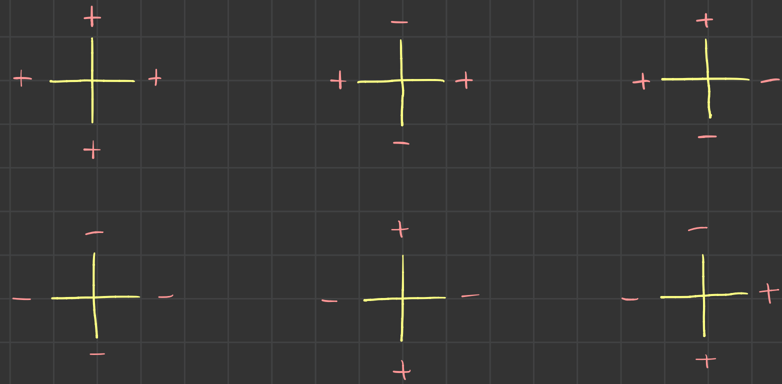
Pf.

$$\begin{aligned} R''_{0\bar{0}} \mathcal{T}_0 \mathcal{T}'_{\bar{0}} &= R''_{0\bar{0}} \cdot R_{01} \cdots R_{0N} \cdot R'_{\bar{0}1} \cdots R'_{\bar{0}N} \\ &= R'_{\bar{0}1} \cdot R_{01} \cdot R''_{0\bar{0}} \cdot R_{02} \cdots R_{0N} \cdot R'_{\bar{0}2} \cdots R'_{\bar{0}N} \\ &= \cdots \\ &= \mathcal{T}'_{\bar{0}} \mathcal{T}_0 R''_{0\bar{0}} \end{aligned}$$

$\square$

Six vertex model (ice model: crystal with hydrogen bonds)

allowed configurations



weights $\quad a \quad b \quad c$

Lemma (YBE) $\Longrightarrow$ $$\frac{a^2+b^2-c^2}{2ab} = \frac{a'^2+b'^2-c'^2}{2a'b'}$$

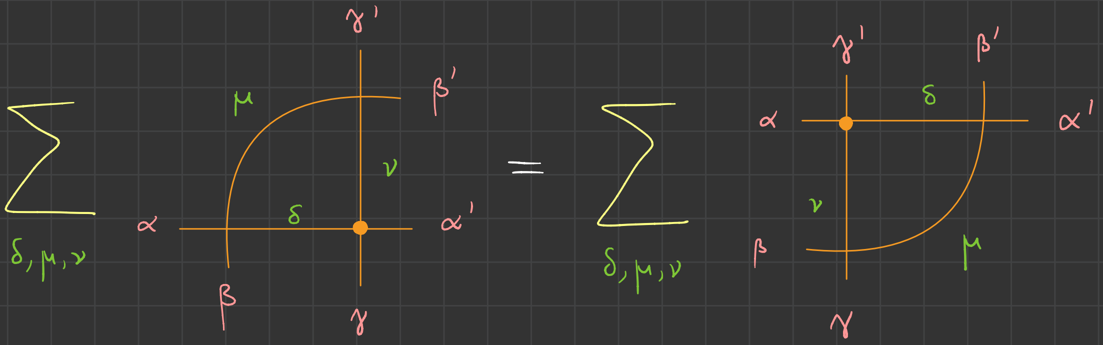
(YBE) $\equiv$ 64 equations

$\forall \alpha, \beta, \gamma, \alpha', \beta', \gamma' \in \{\pm\}$

$$\sum_{\delta,\mu,\nu} R''(\alpha\beta|\delta\mu)\,R(\delta\gamma|\alpha'\nu)\,R'(\mu\nu|\beta'\gamma') =$$

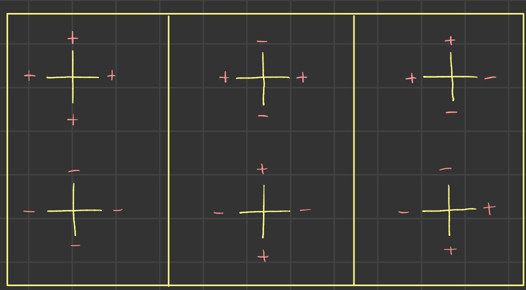
$$= \sum_{\delta,\mu,\gamma} R'(\beta\gamma|\mu\nu)\,R(\alpha\nu|\delta\gamma')\,R''(\delta\mu|\alpha'\beta')$$

Pictorial representation of (YBE)



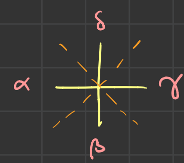
$\bullet = R$

We use the extra symmetries of 6v model to simplify (YBE):



⇝ sign symmetry

⇝ diagonal symmetry

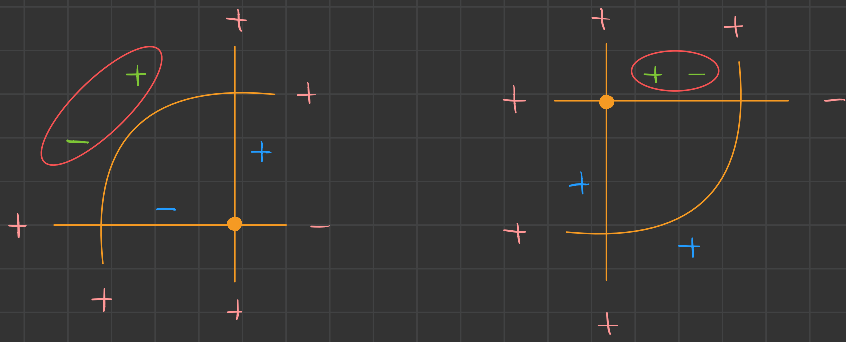


$$R(\alpha\beta|\gamma\delta) = R(\delta\gamma|\beta\alpha)$$
$$R(\alpha\beta|\gamma\delta) = R(\beta\alpha|\delta\gamma)$$
$\Longrightarrow$ eq. trivial for $\alpha = \alpha'$, $\beta = \beta'$, $\gamma = \gamma'$

$$R(\alpha\beta|\gamma\delta) = R(\bar{\alpha}\bar{\beta}|\bar{\gamma}\bar{\delta}) \rightsquigarrow 28 \text{ eq.}$$

In fact, in many cases these reduce to $0 = 0$.

ex.



In fact $\alpha = \beta = \gamma$ is never allowed (-7 eq.)

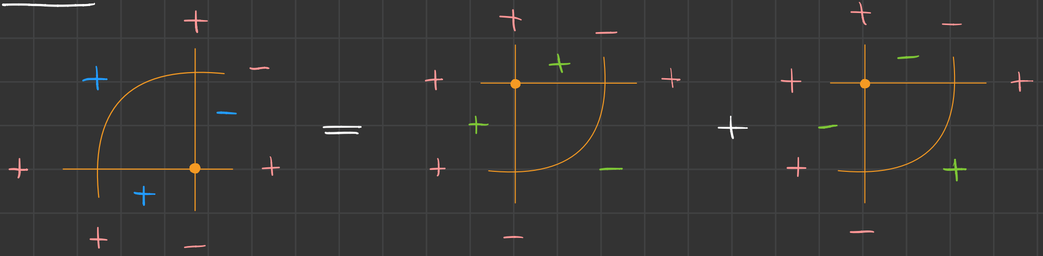
and similarly $\alpha' = \beta' = \gamma'$ (-6 eq.).

Eventually, we get that

$$\{\alpha, \beta, \gamma\} \neq \{\alpha', \beta', \gamma'\} \implies 0 = 0$$

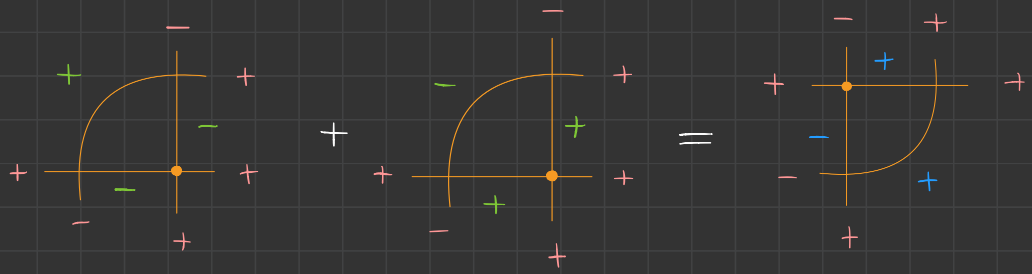
and we are left with only 6 equations, which come in pairs.

Case 1



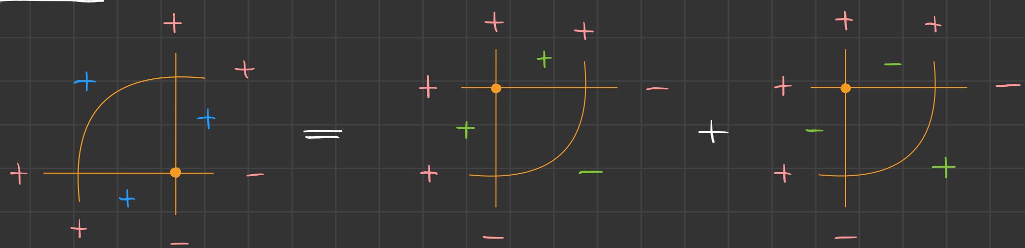
$$a'' b c' = c' a b'' + b' c c''$$

Case 1 (bis)



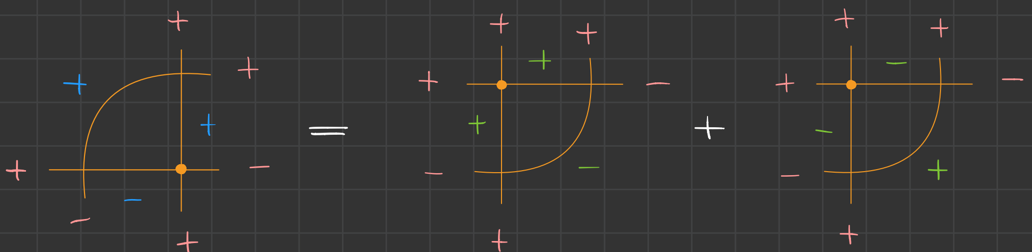
$$c''c\,b' + b''a\,b' = c'\,b\,a''$$

Case 2 (also $+ - - - - +$)



$$a''c\,a' = c'a\,c'' + b'c\,b''$$

Case 3 (also $+ - - - + -$)



$$c''\,b\,a' = b'a\,c'' + c'c\,b''$$

Eliminate a'', b'', c'' :

1) $a''bc' = c'ab'' + b'cc''$

2) $a''ca' = c'ac'' + b'cb''$

3) $c''ba' = b'ac'' + c'cb''$

$$(1)+(2) \qquad \frac{c'ab'' + b'cc''}{bc'} = a'' = \frac{c'ac'' + b'cb''}{ca'}$$

$$\Longrightarrow \quad b''(aa'cc' - bb'cc') = c''(abc'^2 - a'b'c^2)$$

$$(3) \quad b''cc' = c''(a'b - ab')$$

$$\Longrightarrow \quad (a'b - ab')(aa' - bb') = (abc'^2 - a'b'c^2)$$

$$aba'^2 - a'b'b^2 - a^2a'b' + abb'^2 = abc'^2 - a'b'c^2$$

$$\Longrightarrow \quad ab(a'^2 + b'^2 - c'^2) = a'b'(a^2 + b^2 - c^2)$$

$\square$

Rational solutions of (YBE)

Set $\Delta := \dfrac{a^2+b^2-c^2}{2ab} = 1 \rightsquigarrow (a-b)^2 = c^2$

Fix $c = \hbar \in \mathbb{C}^\times$, $b = u$ (parameter), $a = u+\hbar$ and

$$R(u) = \begin{bmatrix} u+\hbar & 0 & 0 & 0 \\ 0 & u & \hbar & 0 \\ 0 & \hbar & u & 0 \\ 0 & 0 & 0 & u+\hbar \end{bmatrix}$$

$\parallel$

$u + \hbar \cdot$ flip

$$\begin{array}{c|cccc} & ++ & -+ & +- & -- \\ \hline ++ & a & 0 & 0 & 0 \\ -+ & 0 & b & c & 0 \\ +- & 0 & c & b & 0 \\ -- & 0 & 0 & 0 & a \end{array}$$

(YBE)
$$R_{12}(u)\, R_{13}(u+v)\, R_{23}(v) = $$
$$= R_{23}(v)\, R_{13}(u+v)\, R_{12}(u)$$

$$\Longrightarrow \quad R_{12}(u-v)\, T_1(u)\, T_2(v) =$$
$$= T_2(v)\, T_1(u)\, R_{12}(u-v)$$

$$\Longrightarrow \quad [T(u), T(v)] = 0$$

The rational RTT algebra

Let $\mathcal{Y}$ be the algebra with generators

$$t_{ij}^{(r)} \qquad r \in \mathbb{Z}_{\geq 0} \ , \ 1 \leq i,j \leq 2$$

$$t_{ij}(u) := \delta_{ij} + \hbar \sum_{r \geq 0} t_{ij}^{(r)} u^{-r-1}$$

$$T(u) := \left[t_{ij}(u) \right]_{1 \leq i,j \leq 2} \in \mathrm{End}(\mathbb{C}^2) \otimes \mathcal{Y}[[u^{-1}]]$$

and relations

$$R_{12}(u-v) T_1(u) T_2(v) = T_2(v) T_1(u) R_{12}(u-v)$$

in $\mathrm{End}(\mathbb{C}^2 \otimes \mathbb{C}^2) \otimes \mathcal{Y}[[u^{-1}, v^{-1}]]$

Unfolded relations

elementary matrix

$$R(u) = u + \hbar \sum_{ij} e_{ij} \otimes e_{ji} \ , \quad T(u) = \sum_{ij} e_{ij} \otimes t_{ij}(u)$$

$$R(u)(e_k \otimes e_l) = u(e_k \otimes e_l) + \hbar (e_l \otimes e_k)$$

$$T(u)(e_j) = \sum_i e_i \otimes t_{ij}(u)$$

standard basis $\mathbb{C}^2$

therefore

$$R(u-v)\, T_1(u)\, T_2(v)\, (e_j \otimes e_\ell) =$$

$$= (u-v) \sum_{ik} e_i \otimes e_k \otimes t_{ij}(u)\, t_{k\ell}(v)$$

$$+ \hbar \sum_{ik} e_k \otimes e_i \otimes t_{ij}(u)\, t_{k\ell}(v)$$

$$T_2(v)\, T_1(u)\, R(u-v)\, (e_j \otimes e_\ell) =$$

$$= (u-v) \sum_{ik} e_i \otimes e_k \otimes t_{k\ell}(v)\, t_{ij}(u)$$

$$+ \hbar \sum_{ik} e_i \otimes e_k \otimes t_{kj}(v)\, t_{i\ell}(u)$$

$$[\, t_{ij}(u),\, t_{k\ell}(v) \,] =$$

$$= \frac{\hbar}{u-v} \Big(t_{kj}(u)\, t_{i\ell}(v) - t_{kj}(v)\, t_{i\ell}(u) \Big)$$

... even more $\left(t_{ij}(u) = \delta_{ij} + \hbar \sum_{r \geq 0} t_{ij}^{(r)} u^{-r-1} \right)$

l.h.s. $(u-v)\left[t_{ij}(u), t_{kl}(v) \right]$

$$(u-v) \cdot \hbar^2 \cdot \sum_{r,s \geq 0} \left[t_{ij}^{(r)}, t_{kl}^{(s)} \right] u^{-r-1} \cdot v^{-s-1} =$$

$$= \hbar^2 \sum_{s \geq 0} \left[t_{ij}^{(0)}, t_{kl}^{(s)} \right] v^{-s-1} - \hbar^2 \sum_{r \geq 0} \left[t_{ij}^{(r)}, t_{kl}^{(0)} \right] u^{-r-1}$$

$$+ \hbar^2 \sum_{r,s \geq 0} \left(\left[t_{ij}^{(r+1)}, t_{kl}^{(s)} \right] - \left[t_{ij}^{(r)}, t_{kl}^{(s+1)} \right] \right) u^{-r-1} v^{-s-1}$$

r.h.s. $\hbar \left(t_{kj}(u) t_{il}(v) - t_{kj}(v) t_{il}(u) \right)$

$$\delta_{kj} \cdot \hbar^2 \cdot \sum_{s \geq 0} t_{il}^{(s)} v^{-s-1} + \delta_{il} \cdot \hbar^2 \cdot \sum_{r \geq 0} t_{kj}^{(r)} u^{-r-1}$$

$$+ \hbar^3 \sum_{r,s \geq 0} t_{kj}^{(r)} t_{il}^{(s)} u^{-r-1} v^{-s-1}$$

$$- \delta_{kj} \cdot \hbar^2 \cdot \sum_{r \geq 0} t_{il}^{(r)} u^{-r-1} - \delta_{il} \cdot \hbar^2 \cdot \sum_{s \geq 0} t_{kj}^{(s)} v^{-s-1}$$

$$- \hbar^3 \sum_{r,s \geq 0} t_{kj}^{(s)} t_{il}^{(r)} u^{-r-1} v^{-s-1}$$

We obtain

$$[t_{ij}^{(0)}, t_{kl}^{(r)}] = \delta_{kj} t_{il}^{(r)} - \delta_{il} t_{kj}^{(r)} \qquad (*)$$

and

$$[t_{ij}^{(r+1)}, t_{kl}^{(s)}] - [t_{ij}^{(r)}, t_{kl}^{(s+1)}] =$$

$$= \hbar \left(t_{kj}^{(r)} t_{il}^{(s)} - t_{kj}^{(s)} t_{il}^{(r)} \right)$$

What kind of beast is Y?

$$\begin{array}{ccccc} U\mathfrak{gl}_2 & \overset{(*)}{\hookrightarrow} & Y & \xrightarrow{ev_0} & U\mathfrak{gl}_2 \\ e_{ij} & \longmapsto & t_{ij}^{(0)} & & \end{array}$$

$$t_{ij}(u) \longmapsto \delta_{ij} + \frac{\hbar}{u} e_{ij}$$

Rmk note that for any $c \in \mathbb{C}^\times$ we get a shift automorphism $\tau_c : Y \longrightarrow Y$

$$T(u) \longmapsto T(u-c)$$

$$\frac{1}{u-c} = \sum_{r \geq 0} c^r u^{-r-1}$$

$$ev_c := ev_0 \circ \tau_c$$

In fact, one checks that (see next page)

$$[t_{ij}^{(r)}, t_{kl}^{(s)}] = \delta_{kj} t_{il}^{(r+s)} - \delta_{il} t_{kj}^{(r+s)} + O(\hbar)$$

$\Longrightarrow$ Y is a $\hbar$-deformation of $U(\mathfrak{gl}_2[t])$ (the Yangian of $\mathfrak{gl}_2$): $t_{ij}^{(r)} \leftrightsquigarrow e_{ij} \otimes t^r$

Set $$q\mathrm{Det}(T(u)) := t_{11}(u)\, t_{22}(u-\hbar) - t_{21}(u)\, t_{12}(u-\hbar)$$

then $$Y_\hbar \mathfrak{sl}_2 := Y_\hbar \mathfrak{gl}_2 \big/ q\mathrm{Det}(T(u)) = 1$$

We won't use much the RTT presentation of Y.

Instead, we consider the Gauss decomposition

$$T(u) = \begin{bmatrix} 1 & 0 \\ x^-(u) & 1 \end{bmatrix} \begin{bmatrix} \xi_1(u) & 0 \\ 0 & \xi_2(u) \end{bmatrix} \begin{bmatrix} 1 & x^+(u) \\ 0 & 1 \end{bmatrix}$$

$\rightsquigarrow$ Drinfeld's new presentation.

Deformation of currents

$$\frac{1}{u-v} = \sum_{p\geq 0} u^{-p-1} \cdot v^{p}$$

$$\hbar^2 \sum_{r,s\geq 0} [t_{ij}^{(r)}, t_{kl}^{(s)}] u^{-r-1} \cdot v^{-s-1} =$$

$$= \hbar^2 \cdot \sum_{p,n\geq 0} \left(\delta_{kj} \cdot t_{il}^{(n)} - \delta_{il} \cdot t_{kj}^{(n)} \right) u^{-p-1} v^{-n-1+p} \quad (1)$$

$$+ \hbar^2 \cdot \sum_{p,m\geq 0} \left(\delta_{il} \cdot t_{kj}^{(m)} - \delta_{kj} \cdot t_{il}^{(m)} \right) u^{-m-1-p-1} v^{p} \quad (2)$$

$$+ \hbar^3 \sum_{p,m,n\geq 0} \left(t_{kj}^{(m)} t_{il}^{(n)} - t_{kj}^{(n)} t_{il}^{(m)} \right) u^{-m-1-p-1} v^{-n-1+p} \quad (3)$$

thus we should consider two types of monomials for $r,s\geq 0$: $u^{-r-1} \cdot v^{s}$ and $u^{-r-1} \cdot v^{-s-1}$.

Note that the contribution of $u^{-r-1} \cdot v^{s}$ is zero.

$$\left.\begin{array}{ll} (1) & p = r, \; n = r-s-1 \\ (2) & p = s, \; m = r-s-1 \end{array}\right\} \text{ cancel each other}$$

$$\left.\begin{array}{ll} (3) & m = r-1-p \\ & n = p-s-1 \end{array}\right\} \quad s+1 \leq p \leq r-1$$

$$\sum_{p=s+1}^{r-1} t_{kj}^{(r-1-p)} \, t_{il}^{(p-s-1)} = \sum_{p=s+1}^{r-1} t_{kj}^{(p-s-1)} \, t_{il}^{(r-1-p)}$$

Instead, for $u^{-r-1} \cdot v^{-s-1}$, we get

(1) $p = r$, $n = r+s$: $\delta_{kj} \cdot t_{il}^{(r+s)} - \delta_{il} \cdot t_{kj}^{(r+s)}$

(2) no contribution

(3) $m = r-1-p$, $n = s+p = r+s-1-m$

$$\hbar \sum_{a=1}^{r} \left(t_{kj}^{(a-1)} \, t_{il}^{(r+s-a)} - t_{kj}^{(r+s-a)} \, t_{il}^{(a-1)} \right)$$

Note that if $r > s$ we have

$$\sum_{a=s+1}^{r} \left(t_{kj}^{(a-1)} \, t_{il}^{(r+s-a)} - t_{kj}^{(r+s-a)} \, t_{il}^{(a-1)} \right) = 0$$

Finally, we get

$$[t_{ij}^{(r)}, t_{kl}^{(s)}] = \delta_{kj} \, t_{il}^{(r+s)} - \delta_{il} \, t_{kj}^{(r+s)} + $$

$$+ \hbar \sum_{a=1}^{\min(r,s)} \left(t_{kj}^{(a-1)} \, t_{il}^{(r+s-a)} - t_{kj}^{(r+s-a)} \, t_{il}^{(a-1)} \right)$$

Therefore, we can think of $Y_\hbar \mathfrak{gl}_2$ as a flat deformation of the current algebra

$$Y_\hbar \mathfrak{gl}_2 \Big|_{\hbar=0} = U(\mathfrak{gl}_2[t])$$

Note also that, for any $h, h' \in \mathbb{C}^\times$, $Y_h \mathfrak{gl}_2 \simeq Y_{h'} \mathfrak{gl}_2$.

Indeed, since

$$[t_{ij}^{(r)}, t_{kl}^{(s)}] = \delta_{kj} t_{il}^{(r+s)} - \delta_{il} t_{kj}^{(r+s)} +$$

$$+ \hbar \sum_{a=1}^{\min(r,s)} \left(t_{kj}^{(a-1)} t_{il}^{(r+s-a)} - t_{kj}^{(r+s-a)} t_{il}^{(a-1)} \right)$$

we get an isomorphism $Y_{h'} \mathfrak{gl}_2 \longrightarrow Y_h \mathfrak{gl}_2$ by rescaling

$$t_{ij}^{(r)} \longmapsto (h'/h)^r \, t_{ij}^{(r)}$$

In particular, $Y_h \mathfrak{gl}_2 \simeq Y_1 \mathfrak{gl}_2 \not\simeq U(\mathfrak{gl}_2[t])$.

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RTT presentation for $\mathfrak{g}$ simple ($\mathfrak{g} \neq \mathfrak{e}_8$)

C. Wendlandt

The R-matrix presentation for the Yangian of a simple Lie algebra (2018)

$V \in$ f.d. rep of $Y(\mathfrak{g}) \rightsquigarrow$ there is an RTT algebra $\mathcal{X}_V(\mathfrak{g}) \twoheadrightarrow Y(\mathfrak{g})$