R-matrix

Assumptions: $\underbrace{A \subset T}_{\text{tori}} \subset \overset{\text{reductive group}}{G}$ acting on X of the previous lessons.

"$H_G^*(-) \simeq H_T^*(-)^W$"

Then, $\exists !$

$$\mathrm{Stab}_{C,\varepsilon} : H_G^*(X^A) \longrightarrow H_G^*(X)$$

After inverting $e(N_-(Z))$ for Z connected component of X^A, $\exists$ $\mathrm{Stab}_{C,\varepsilon}^{-1}$

Fix a polarization ε.

Definition $R_{C',C} := \mathrm{Stab}_{C'}^{-1} \circ \mathrm{Stab}_C \in \mathrm{End}\left(H_G^*(X^A) \otimes \mathbb{Q}(\underset{\text{Lie}(G)}{\underset{\parallel}{\mathfrak{g}}}) \right)$

Braid Relations

Let $\mathcal{F} \subset C$ be a codimension 2 facet and let

$$C = C_0, C_1, \dots, C_{2n} = C$$

be the chambers containing $\mathcal{F}$ as a facet, in cyclic order around $\mathcal{F}$. Then

$$R_{C_0,C_1} R_{C_1,C_2} \cdots R_{C_{2n-1},C_{2n}} = \mathrm{id}$$

<u>Remark</u> For X = quiver variety :

Braid Relations $\Longrightarrow$ Yang-Baxter equations

Matrix Elements

Let Z be a connected component of X^A, which is minimal w.r.t. $<_C$ for some chamber C.

<u>Theorem</u> $\left(R_{-C,C}\cdot \gamma_1, \gamma_2\right) = \int_Z \gamma_1 \cup \gamma_2 \cup \frac{e(N_+(Z)\otimes \hbar)}{e(N_+(Z))}$ for $\gamma_i \in H_G^*(Z)$.

<u>Remark</u> For non-minimal Z, one has other contributions to consider.

Root R-matrices and symplectic resolutions

<u>Definition</u> (Beauville)

A normal complex variety Y is said to have <u>symplectic singularities</u> if

1. $\exists$ a symplectic form ω defined on the smooth locus Y^{sm} of Y; and
2. if $\pi : X \longrightarrow Y$ is a resolution of singularities (i.e. a birational proper morphism from a smooth complex variety), then $\pi^* \omega$ is a (possibly degenerate) symplectic form on X.

If in addition, $\pi^* \omega$ is non-degenerate, X is said to be a symplectic resolution.

In this part, we assume that X is a symplectic resolution.

Let α a root and consider the hypeplane $\alpha^\perp$.
Let C and C' be two chambers separated by $\alpha^\perp$. Assume that $\alpha(C) > 0$.
Set

$$R_\alpha := R_{C', C}$$

<u>Proposition</u> $R_\alpha = 1 + \frac{\hbar}{\alpha} r_\alpha + O(\alpha^{-2})$

$r_\alpha \in \mathrm{End}(H_G^*(X^A))$

<u>Remark</u> $\exists$ Lagrangian cycle τ_α on $X^A \times X^A$ such that $r_\alpha = \Theta_{\tau_\alpha}$.